Calculus I Portfolio

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Problem #1 - Section 1.2 - Exercise 2
Collaborators: Include the (first and last) names of your group members for projects

Let $g(x) = -\dfrac{|x+3|}{x+3}$.

(a) What is the domain of g?

Solution: The domain of g is all real numbers except $x = -3$, or equivalently $(-\infty, -3) \cup (-3, \infty)$. If we wanted to compute $g(-3)$ the denominator of the fraction would evaluate to 0. Outputs for any other real number input can be computed, and therefore those numbers are elements of the domain.

(b) Use a sequence of values near $a = -3$ to estimate the value of $\lim_{x \to -3} g(x)$, if you think the limit exists. If you think the limit doesn't exist, explain why.

x	$g(x)$
-3.1	1
-3.01	1
-3.001	1
-3.0001	1
-2.9999	-1
-2.999	-1
-2.99	-1
-2.9	-1

We can use a computer or calculator generate the table, or make the computations by hand. It appears from the table that the limit does not exist. Using inputs smaller than -3, we can make g as close to 1 (actually, exactly 1) that we wish. Similarly, by using inputs larger than -3, we can make g as close to -1 as we wish. Since the function g cannot have two different limits at $x = -3$, we say the limit does not exist.

(c) Use algebra to simplify the expression $\frac{|x+3|}{x+3}$ and hence work to evaluate $\lim_{x\to -3} g(x)$ exactly, if it exists, or to explain how your work shows the limit fails to exist. Discuss how your findings compare to your results in (b).

Solution: Since the absolute value function is defined piece-wise, we will perform two different sets of calculations.

If $x > -3$, then $x + 3 > 0$ and

$$\frac{|x+3|}{x+3} = \frac{x+3}{x+3} = 1.$$

If $x < -3$, then $x + 3 < 0$ and

$$\frac{|x+3|}{x+3} = \frac{-(x+3)}{x+3} = -1.$$

These computations algebraically show that our chart from part (b) is not misleading. For $x < -3$, $g(x) = 1$ and for $x > -3$, $g(x) = -1$. Since these are distinct values, the limit of g as x approaches -3 does not exist.

(d) True or false: $g(-3) = -1$. Why?

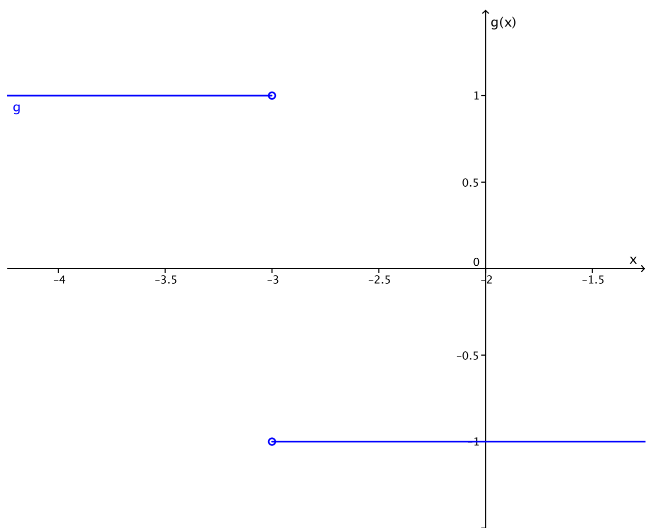
Solution: This statement is false. Since -3 is not in the domain of g, the left side of the equation does not make sense. If we did evaluate g at $x = -3$, we would arrive at $\frac{0}{0}$, an indeterminate form.

(e) True or false: $-\frac{|x+3|}{x+3} = -1$. Why? How is this equality connected to your work above with the function g?

This statement is also false. The equation holds only when $x > -3$. The expression on the left hand side is that of the function g, when it makes sense. Since the statement is false, this implies that the function g has a discontinuity at $x = -3$.

(f) Based on all of your work above, construct an accurate, labeled graph of $y = g(x)$ on the interval $[-4, -2]$, and write a sentence that explains what you now know about $\lim_{x \to -3} g(x)$.

Solution: The following graph shows the attributes of g determined above. Namely, that -3 is not in the domain, g is discontinuous at $x = -3$, and $\lim_{x \to -3} g(x)$ does not exist.



Notice that the y-axis crosses at $x = -2$, only so that it will appear in the picture.