MAT 2550 Hw due 2/26 Myles Miller

1.

Proposition. *Prove that if $A : V \to W$ is an isomorphism and $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, ..., \boldsymbol{v}_n$ is a basis for V, then $A\boldsymbol{v}_1, A\boldsymbol{v}_2, A\boldsymbol{v}_3, ..., A\boldsymbol{v}_n$ is a basis in W*

Proof. Proving by contradiction.

Assuming that the basis of W is contains some addition vector Av_{n+1}, we know that, by performing the reverse of the isomorphism, we will get v_{n+1}, a vector unreachable through the basis of V, thus, since it is not in V's basis, is not in V.

Therefore, W's basis contains no extra vectors.

Assuming that the basis of W is smaller than V's, such that some Av_i is reachable through the other vectors in W's basis.

Performing the inverse isomorphism, we result in v_i, which is in v's basis.

This is a contradiction, so we know that W's basis is not smaller than V's.

Given that we assumed the two to be isomorphic, and their basis' are of the same size, we know that their basis' are isomorphic, so $A\mathbf{v}_1, A\mathbf{v}_2, A\mathbf{v}_3, ..., A\mathbf{v}_n$ forms W's basis. □

2.

Proposition. *Find all right inverses to the 1×2 matrix $A = (1, 1)$. Show that there is no left inverse.*

Proof. We know that the matrix we are looking for results the following:

$$A * A_r^{-1} = 1$$

Given that we know that $A = (1, 1)$, we know that A_r^{-1} looks like $\begin{bmatrix} x \\ y \end{bmatrix}$

Thus:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = [x * 1 + y * 1] = [1]$$

Therefore, we know that $x + y = 1$. In efforts to eliminate variables, we recognize that $y = 1 - x$, so $A_r^{-1} = \begin{bmatrix} x \\ 1 - x \end{bmatrix}$

Assuming there to be some A_L^{-1} such that $A_L^{-1} \times A = 1$, we know that A_L^{-1} is no taller than 1 and no wider than 1, so that we may multiply the two.

Thus, $A_L^{-1} = \begin{bmatrix} x \end{bmatrix}$.

However, $\begin{bmatrix} x \end{bmatrix} \times \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} x & x \end{bmatrix} \neq \begin{bmatrix} 1 \end{bmatrix}$

□

3.

Proposition. *Find all left inverses of* $\begin{bmatrix}1 & 2 & 3\end{bmatrix}^T$

Proof. Letting $A = \begin{bmatrix}1\\2\\3\end{bmatrix}$

We know that $A_L^{-1} \times A = 1$, which means that A_L^{-1} looks like $\begin{bmatrix}x & y & z\end{bmatrix}$

That means:$A_L^{-1} \times A = \begin{bmatrix}x & y & z\end{bmatrix} \times \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}x * 1 + 2 * y + 3 * z\end{bmatrix} = 1$

Which means that $x+2y+3z = 1$, so $x = 1-2y-3z$. thus $A_L^{-1} = \begin{bmatrix}1-2y-3z & y & z\end{bmatrix}$

Proving this:

$$\begin{bmatrix}1-2y-3z & y & z\end{bmatrix} \times \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}1*(1-2y-3z)+(2*y)+(3*z)\end{bmatrix} =$$

$$\begin{bmatrix}1-2y-3z+2y+3z\end{bmatrix} = \begin{bmatrix}1\end{bmatrix}$$

□

4.

Proposition. *Is the column* $(1, 2.3)^T$ *invertible?*

Proof. Letting $A_R^{-1} = \begin{bmatrix}x\end{bmatrix}$, the only properly sized matrix which can be multiplied with A in that order.

This results $A \times A_R^{-1} = \begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}x\end{bmatrix} = \begin{bmatrix}x\\2x\\3x\end{bmatrix} \neq \begin{bmatrix}1\end{bmatrix}$ □

6.

Proposition. *Suppose the product AB is invertible. Show that A is right invertible and B is left invertible.*

Proof. □

7.

Proposition. *Prove that i*

Proof. □

13.

Proposition. *Prove that i*

Proof. □

1.

Proposition. *Prove that i*

Proof. □

3.

Proposition. *Prove that i*

Proof. □

$$\subset, \subseteq, \supset, \supseteq, \cup, \cap, \in, \notin$$