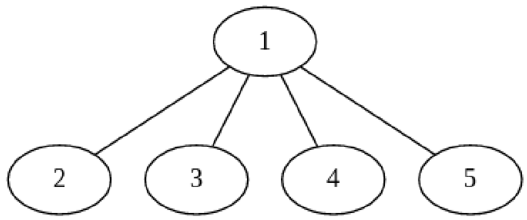
For every connected undirected graph G, there exists a function s_G with two parameters n and m defined like so:

$s_G(n, m)$ = Maximum number of distinct, connected subgraphs of G of order n, in which each vertex of G is used in at most m of these subgraphs.
For example, in this graph:



$s(1, 1) = |V| = 5$
$s(2, \infty) = |E| = 4$
$s(2, 1) = 1$ (in fact for all diameter-2 graphs, $s(2, m) = m$ up to $|E|$)
Another useful property:
$s(|V| - 1, \infty)$ = number of non-articulation points in the graph.

My question is, does this s_G function unique determine graph G?
In other words, are two graphs G and G' isomorphic if and only if they have the same function?
And if so, if you restrict the second parameter m to the two values of $\{1, \infty\}$ does this new function also do the same?